

Sequenced Units for the Common Core State Standards in Mathematics

Grade 4

In the years prior to Grade 4, students gained an understanding of multiplication and division of whole numbers, generalized strategies for addition and subtraction to multi-digit numbers, developed understanding of fractions as numbers, and reasoned with shapes and their attributes. They used arrays and the concept of area to develop computational strategies for multiplication and division.

Throughout Grade 4, students continue to develop their understanding of number. They generalize their understanding of place value to 1,000,000. Students extend their understanding of the four operations to include multiplicative compare problems, operations with multi-digit numbers, and multiplying fractions by whole numbers. Students further develop their understanding of fractions to include addition of fractions with like denominators and comparison and ordering of fractions with either like numerators or like denominators. The geometry focus in Grade 4 is on reasoning about angle measurement and lines.

This document reflects our current thinking related to the intent of the Common Core State Standards for Mathematics (CCSSM) and assumes 160 days for instruction, divided among 14 units. The number of days suggested for each unit assumes 45-minute class periods and is included to convey how instructional time should be balanced across the year. The units are sequenced in a way that we believe best develops and connects the mathematical content described in the CCSSM; however, the order of the standards included in any unit does not imply a sequence of content within that unit. Some standards may be revisited several times during the course; others may be only partially addressed in different units, depending on the focus of the unit. Strikethroughs in the text of the standards are used in some cases in an attempt to convey that focus, and comments are included throughout the document to clarify and provide additional background for each unit.

Throughout Grade 4, students should continue to develop proficiency with the Common Core's eight Standards for Mathematical Practice:

- 1. Make sense of problems and persevere in solving them.**
- 2. Reason abstractly and quantitatively.**
- 3. Construct viable arguments and critique the reasoning of others.**
- 4. Model with mathematics.**
- 5. Use appropriate tools strategically.**
- 6. Attend to precision.**
- 7. Look for and make use of structure.**
- 8. Look for and express regularity in repeated reasoning.**

These practices should become the natural way in which students come to understand and do mathematics. While, depending on the content to be understood or on the problem to be solved, any practice might be brought to bear, some practices may prove more useful than others. Opportunities for highlighting certain practices are indicated in different units in this document, but this highlighting should not be interpreted to mean that other practices should be neglected in those units.

When using this document to help in planning your district's instructional program, you will also need to refer to the CCSSM document, relevant progressions documents for the CCSSM, and the appropriate assessment consortium framework.

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Unit 1: Exploring multiples and factors	Suggested number of days: 10
<p>In this unit students develop understanding of multiples and factors, applying their understanding of multiplication from the previous year. This understanding lays a strong foundation for generalizing strategies learned in previous grades to develop, discuss, and use efficient, accurate, and generalizable computational strategies involving multi-digit numbers. These concepts and the terms “prime” and “composite” are new to Grade 4, so they are introduced early in the year to give students ample time to develop and apply this understanding.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA</p> <p>B. Gain familiarity with factors and multiples.</p> <p>4. Find all factor pairs for a whole number in the range 1–100. Recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.</p> <p>C. Generate and analyze patterns.</p> <p>5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p> <p>Common Core State Standards for Mathematical Practice</p> <p>3. Construct viable arguments and critique the reasoning of others.</p> <p>7. Look for and make use of structure.</p>	<p>Comments</p> <p>While working on 4.OA.C.5, students use manipulatives to determine whether a number is prime or composite. Although there are shape patterns in arrays, the focus of this unit is number patterns. 4.OA.C.5 is repeated in unit 13, where the focus will be on identifying shape patterns.</p> <p>The focus of this unit is not necessarily to become fluent in finding all factor pairs, but to use student’s understanding of the concept and language to discuss the structure of multiples and factors (MP.3, MP.7).</p>

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Unit 2: Using multiplication and division strategies with larger numbers	Suggested number of days: 15
<p>In this unit students continue using computational and problem-solving strategies, with a focus on building conceptual understanding of multiplication of larger numbers and division with remainders. Area and perimeter of rectangles provide one context for developing such understanding.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA</p> <p>A. Use the four operations with whole numbers to solve problems.</p> <p>3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p>Number and Operations in Base Ten² — 4.NBT</p> <p>B. Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>NOTE: ²Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p> <p>Measurement and Data — 4.MD</p> <p>A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>3. Apply the area and perimeter formulas for rectangles in real world and mathematical problems. <i>For example, find the width of a rectangular room given the area of the flooring and the length, by viewing the area formula as a multiplication equation with an unknown factor.</i></p> <p>Common Core State Standards for Mathematical Practice</p> <p>1. Make sense of problems and persevere in solving them.</p> <p>2. Reason abstractly and quantitatively.</p> <p>8. Look for and express regularity in repeated reasoning.</p>	<p>Comments</p> <p>4.OA.A.3 is the first time students are expected to interpret remainders based upon the context. All four operations will be addressed in unit 8, and the standard will be finalized in unit 14.</p> <p>4.MD.A.3 provides the context of area and perimeter of rectangles to use for problem solving. Students are first introduced to formulas in this unit and make sense of the formulas using their prior work with area and perimeter.</p> <p>Students make sense of multi-step problems (MP.1) and reason about how the formulas connect to the context (MP.2). The use of generalized strategies and formulas provides an opportunity to investigate and use regularity in repeated reasoning (MP.8).</p>

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Unit 3: Decomposing and composing fractions for addition and subtraction	Suggested number of days: 8
<p>In this unit students extend their prior knowledge of unit fractions with denominators of 2, 3, 4, 6, and 8 from Grade 3 to include denominators of 5, 10, 12, and 100. In Grade 4, they use their understanding of partitioning to find unit fractions to compose and decompose fractions in order to add fractions with like denominators. This is foundational for further work with fractions later in the year, such as comparing fractions and multiplying fractions by a whole number.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Number and Operations—Fractions³ — 4.NF</p> <p>B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</p> <p>3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p> <p style="padding-left: 20px;">a. Understand addition and subtraction of fractions as joining and separating parts referring to the same whole.</p> <p style="padding-left: 20px;">b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation. Justify decompositions, e.g., by using a visual fraction model. <i>Examples:</i> $3/8 = 1/8 + 1/8 + 1/8$; $3/8 = 1/8 + 2/8$; $2 \frac{1}{8} = 1 + 1 + 1/8 = 8/8 + 8/8 + 1/8$.</p> <p>NOTE: ³Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Common Core State Standards for Mathematical Practice</p> <p>4. Model with mathematics.</p>	<p>Comments</p> <p>Students use visual and concrete models to represent a fractional situation in order to add and subtract fractions (MP.4).</p>

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Unit 4: Applying place value concepts in whole number addition and subtraction	Suggested number of days: 12
<p>The focus of this unit is to provide students time to develop and practice efficient addition and subtraction of multi-digit whole numbers while developing place value concepts.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Number and Operations in Base Ten² — 4.NBT</p> <p>A. Generalize place value understanding for multi-digit whole numbers.</p> <ol style="list-style-type: none"> 1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i> 2. Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using $>$, $=$, and $<$ symbols to record the results of comparisons. 3. Use place value understanding to round multi-digit whole numbers to any place. <p>B. Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <ol style="list-style-type: none"> 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm. <p>NOTE: ²Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p> <p>Common Core State Standards for Mathematical Practice</p> <ol style="list-style-type: none"> 6. Attend to precision. 8. Look for and express regularity in repeated reasoning. 	<p>Comments</p> <p>4.NBT.A.1 will be revisited in unit 6 connected to conversions within the metric system of measurement.</p> <p>4.NBT.A.3 will be revisited in unit 7 with multiplication and division as a context.</p> <p>4.NBT.B.4 will be revisited in unit 8 and finalized in unit 14 for fluency in addition and subtraction of multi-digit whole numbers.</p> <p>Students use the structure of the base-ten system to generalize their strategies and to discuss reasonableness of their computations and work towards fluency (MP.6, MP.8).</p>

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Unit 5: Understanding fraction equivalence and comparison	Suggested number of days: 12
<p>In this unit students develop an understanding of fraction equivalence and various methods for comparing fractions. Students should understand that when comparing fractions, it is not always necessary to generate equivalent fractions. Other methods, such as comparing fractions to a benchmark, can be used to discuss relative sizes. The justification of comparing or generating equivalent fractions using visual models is an emphasis of this unit.¹</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Number and Operations—Fractions³ — 4.NF</p> <p>A. Extend understanding of fraction equivalence and ordering.</p> <ol style="list-style-type: none"> 1. Explain why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions. 2. Compare two fractions with different numerators and different denominators, e.g., by creating common denominators or numerators, or by comparing to a benchmark fraction such as $1/2$. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model. <p>NOTE: ³Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Common Core State Standards for Mathematical Practice</p> <ol style="list-style-type: none"> 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. 	<p>Comments</p> <p>Students justify their methods for generating equivalent fractions and comparing fractions by using their conceptual understanding and models (MP.3, MP.5).</p>

¹ For additional information on equivalent fractions in Grade 4, see pages 5-6 in the Fractions progressions document.

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Unit 6: Introducing measurement conversions	Suggested number of days: 10
<p>In this unit students build a conceptual understanding of the relative sizes of units of measure within a single system of measurement. Measurement conversions are used to introduce multiplication as a comparison. The concepts in this unit are foundational for the concepts in unit 7 and unit 8.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA A. Use the four operations with whole numbers to solve problems. 1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <p>Number and Operations in Base Ten² — 4.NBT A. Generalize place value understanding for multi-digit whole numbers. 1. Recognize that in a multi-digit whole number, a digit in one place represents ten times what it represents in the place to its right. <i>For example, recognize that $700 \div 70 = 10$ by applying concepts of place value and division.</i> NOTE: ²Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p> <p>Measurement and Data — 4.MD A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 1. Know relative sizes of measurement units within one system of units including km, m, cm; kg, g; lb, oz.; l, ml; hr, min, sec. Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a two- column table. <i>For example, know that 1 ft is 12 times as long as 1 in. Express the length of a 4 ft snake as 48 in. Generate a conversion table for feet and inches listing the number pairs (1, 12), (2, 24), (3, 36), ...</i></p> <p>Common Core State Standards for Mathematical Practice 2. Reason abstractly and quantitatively. 6. Attend to precision. 7. Look for and make use of structure.</p>	<p>Comments</p> <p>4.OA.A.1 is repeated in unit 11, in which the focus is on multiplication of fractions.²</p> <p>4.NBT.A.1 was addressed in unit 4, in which the focus was on addition and subtraction. In this unit, metric measurement provides an opportunity to deepen the students' understanding of place value in relation to multiples of 10.</p> <p>4.MD.A.1 introduces units of measure new to Grade 4.</p> <p>In this unit students look for patterns in different measurement systems (MP.2, MP.7) and discuss precisely how many times larger one unit is than another (MP.6)</p>

² For additional information about the connections to this standard, see page 21 in the PARCC Model Content Frameworks.

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Unit 7: Solving problems using multiplicative comparison	Suggested number of days: 10
<p>In this unit students are introduced to multiplicative compare problems, extending their conceptual work with multiplicative comparison from unit 6. For students to develop this concept, they must be provided rich problem situations that encourage them to make sense of the relationships among the quantities involved, model the situation, and check their solution using a different method. CCSSM Table 2 is an important resource for understanding multiplicative comparison problems, which are new to Grade 4 students.³</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA A. Use the four operations with whole numbers to solve problems. 2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹ NOTE: ¹See Glossary, Table 2.</p> <p>Number and Operations in Base Ten² — 4.NBT A. Generalize place value understanding for multi-digit whole numbers. 3. Use place value understanding to round multi-digit whole numbers to any place. NOTE: ²Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000</p> <p>Measurement and Data — 4.MD A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>Common Core State Standards for Mathematical Practice 1. Make sense of problems and persevere in solving them.</p>	<p>Comments</p> <p>4.OA.A.2 is also addressed in unit 14 because of the time it takes to master the concepts and its importance to future mathematics.⁴</p> <p>4.NBT.A.3 was addressed in unit 4 with a focus on addition and subtraction. In this unit, the focus is on multiplication and division.</p> <p>4.MD.A.2 is used as a context for multiplicative compare problems with whole numbers only. This standard is revisited in unit 8 to include the four operations, and addressed in unit 12 with decimal fractions.</p> <p>Students use charts and diagrams to explain their own methods as well make sense of approaches taken by others (MP.1).</p>

³ For additional information on multiplication “compare” problems, see page 29 in the Operations and Algebraic Thinking progressions document and Table 2 on page 89 in the Common Core State Standards for Mathematics.

⁴ For more information on content emphases in Grade 4, see page 22 in the PARCC Model Content Frameworks.

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Unit 8: Solving measurement problems using the four operations	Suggested number of days: 12
<p>In this unit students combine competencies from different domains to solve measurement problems using the four operations.⁵ Measurement is included in this unit to provide a context for problem solving. All of the problem types in Table 1 and Table 2 on pages 88 and 89 of the Common Core State Standards for Mathematics should be addressed in this unit.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA A. Use the four operations with whole numbers to solve problems. 3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p>Number and Operations in Base Ten² — 4.NBT B. Use place value understanding and properties of operations to perform multi-digit arithmetic. 4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>Measurement and Data — 4.MD A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit. 2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>Common Core State Standards for Mathematical Practice</p> <p>1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 6. Attend to precision.</p>	<p>Comments</p> <p>4.OA.A.3 and 4.NBT.B.4 are repeated here to include all four operations and will be finalized in unit 14. Repeating these standards throughout the year provides students multiple opportunities to develop these skills—which are major areas of focus for this grade level.</p> <p>4.MD.A.2 is repeated from the previous unit, but in this unit the emphasis is on using the four operations and all problem types. This standard will be finalized in unit 12 to include decimal fractions.</p> <p>Students use various diagrams and precise language to solve measurement problems and explain their strategies (MP.1, MP.6). They make connections between abstract representations and the problem situations (MP.2).</p>

⁵ For additional information, see page 20 in the Geometric Measurement progressions document.

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Unit 9: Solving addition and subtraction word problems involving fractions and mixed numbers	Suggested number of days: 10
<p>In this unit students will use their understanding of adding and subtracting fractions and generating equivalent fractions to solve problems involving fractions and mixed numbers. Students rely on their previous work with whole numbers as fractions to compose and decompose whole numbers into fractional quantities.⁶ Data is used in this unit to support students' understanding of fractional quantities both smaller and larger than 1.⁷</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Number and Operations—Fractions³ — 4.NF</p> <p>B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.</p> <p>3. Understand a fraction a/b with $a > 1$ as a sum of fractions $1/b$.</p> <p style="padding-left: 20px;">c. Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.</p> <p style="padding-left: 20px;">d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like denominators, e.g., by using visual fraction models and equations to represent the problem.</p> <p>NOTE: ³Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Measurement and Data — 4.MD</p> <p>B. Represent and interpret data.</p> <p>4. Make a line plot to display a data set of measurements in fractions of a unit ($1/2$, $1/4$, $1/8$). Solve problems involving addition and subtraction of fractions by using information presented in line plots. <i>For example, from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection.</i></p> <p>Common Core State Standards for Mathematical Practice</p> <p>2. Reason abstractly and quantitatively.</p> <p>4. Model with mathematics.</p>	<p>Comments</p> <p>4.MD.B.4 extends students' work from Grade 3 with simple fractions on a line plot (3.MD.B.4) to include eighths and to now solve addition and subtraction problems using the data.</p> <p>Students reason about fractions by using abstract models to represent both the data and the fractional quantities (MP.2, MP.4).</p>

⁶ For more information on students' work with mixed numbers, see pages 6-7 in the Fractions progressions document.

⁷ For additional information, see Table 1 on page 4 of the Measurement Data progressions document.

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Unit 10: Angle measurement	Suggested number of days: 12
<p>This unit is an introduction to angles and angle measurement. Students start this unit drawing points, lines, segments, rays and angles since it is foundational to the other standards in this unit. Students use their understanding of equal partitioning and unit measurement to understand angle and turn measure.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Measurement and Data — 4.MD</p> <p>C. Geometric measurement: understand concepts of angle and measure angles.</p> <p>5. Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:</p> <p style="padding-left: 20px;">a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1/360$ of a circle is called a “one-degree angle,” and can be used to measure angles.</p> <p style="padding-left: 20px;">b. An angle that turns through n one-degree angles is said to have an angle measure of n degrees.</p> <p>6. Measure angles in whole-number degrees using a protractor. Sketch angles of specified measure.</p> <p>Geometry — 4.G</p> <p>A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles.</p> <p style="padding-left: 20px;">1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures.</p> <p>Common Core State Standards for Mathematical Practice</p> <p>4. Model with mathematics.</p> <p>5. Use appropriate tools strategically.</p>	<p>Comments</p> <p>In this unit, 4.G.A.1 focuses on drawing points, lines, line segments, rays, and different types of angles. The standard will be addressed in its entirety in unit 13.</p> <p>Students select and use a protractor to measure angles and represent the angles with drawings (MP.4, MP.5).</p>

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Unit 11: Multiplying fractions by whole numbers	Suggested number of days: 12
<p>In this unit students apply their understanding of composing and decomposing fractions to develop a conceptual understanding of multiplication of a fraction by a whole number. Students also use and extend their previous understandings of operations with whole numbers and relate that understanding to fractions.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA A. Use the four operations with whole numbers to solve problems. 1. Interpret a multiplication equation as a comparison, e.g., interpret $35 = 5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5. Represent verbal statements of multiplicative comparisons as multiplication equations.</p> <p>Number and Operations—Fractions³ — 4.NF B. Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers. 4. Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. a. Understand a fraction a/b as a multiple of $1/b$. <i>For example, use a visual fraction model to represent $5/4$ as the product $5 \times (1/4)$, recording the conclusion by the equation $5/4 = 5 \times (1/4)$.</i> b. Understand a multiple of a/b as a multiple of $1/b$, and use this understanding to multiply a fraction by a whole number. <i>For example, use a visual fraction model to express $3 \times (2/5)$ as $6 \times (1/5)$, recognizing this product as $6/5$. (In general, $n \times (a/b) = (n \times a)/b$.)</i> c. Solve word problems involving multiplication of a fraction by a whole number, e.g., by using visual fraction models and equations to represent the problem. <i>For example, if each person at a party will eat $3/8$ of a pound of roast beef, and there will be 5 people at the party, how many pounds of roast beef will be needed? Between what two whole numbers does your answer lie?</i></p> <p>NOTE: ³ Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Common Core State Standards for Mathematical Practice 1. Make sense of problems and persevere in solving them. 6. Attend to precision.</p>	<p>Comments</p> <p>4.OA.A.1 is readdressed in this unit to include multiplication of fractions and apply the understanding of “times as much” (i.e. multiplication as comparison) to multiplying a fraction by a whole number.⁸</p> <p>Students use precise language to communicate their comprehension of the problem situations and defend their various solution methods (MP.1, MP.6)</p>

⁸ For additional information on the connection between these two standards, see page 21 in the PARCC Model Content Frameworks.

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Unit 12: Comparing decimal fractions and understanding notation	Suggested number of days: 10
<p>In this unit of study students use their previous work with fractions to represent special fractions in a new way. Students use their understanding of equivalent fractions to begin to use decimal notation—however, it is not the intent at this grade level to connect this notation to the base-ten system. The focus is on solving word problems involving simple fractions or decimals.⁹ Work with money can support this work with decimal fractions.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Number and Operations—Fractions³ — 4.NF</p> <p>C. Understand decimal notation for fractions, and compare decimal fractions.</p> <p>5. Express a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100.⁴ <i>For example, express $3/10$ as $30/100$, and add $3/10 + 4/100 = 34/100$.</i></p> <p>NOTE:⁴ Students who can generate equivalent fractions can develop strategies for adding fractions with unlike denominators in general. But addition and subtraction with unlike denominators in general is not a requirement at this grade.</p> <p>6. Use decimal notation for fractions with denominators 10 or 100. <i>For example, rewrite 0.62 as $62/100$; describe a length as 0.62 meters; locate 0.62 on a number line diagram.</i></p> <p>7. Compare two decimals to hundredths by reasoning about their size. Recognize that comparisons are valid only when the two decimals refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual model.</p> <p>NOTE:³ Grade 4 expectations in this domain are limited to fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100.</p> <p>Measurement and Data — 4.MD</p> <p>A. Solve problems involving measurement and conversion of measurements from a larger unit to a smaller unit.</p> <p>2. Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals, and problems that require expressing measurements given in a larger unit in terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale.</p> <p>Common Core State Standards for Mathematical Practice</p> <p>3. Construct viable arguments and critique the reasoning of others.</p> <p>7. Look for and make use of structure.</p>	<p>Comments</p> <p>4.MD.A.2 was addressed in unit 7. It is important to note that students are not expected to do computations with quantities in decimal notation. Students can use visual fraction models to solve problems involving simple fractions or decimals.</p> <p>Students compare decimals fractions and justify their comparisons using either a fraction model or their understanding of the notation (MP.3, MP.7).</p>

⁹ For helpful examples and information about decimal fractions, see pages 8-9 in the Fractions progressions document.

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Unit 13: Recognizing and analyzing attributes of 2-dimensional shapes	Suggested number of days: 12
<p>In this unit students develop their spatial reasoning skills by using a wide variety of attributes to talk about 2-dimensional shapes. Students analyze geometric figures based on angle measurement, parallel and perpendicular lines, and symmetry.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA C. Generate and analyze patterns. 5. Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. <i>For example, given the rule “Add 3” and the starting number 1, generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.</i></p> <p>Measurement and Data — 4.MD C. Geometric measurement: understand concepts of angle and measure angles. 7. Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems, e.g., by using an equation with a symbol for the unknown angle measure.</p> <p>Geometry — 4.G A. Draw and identify lines and angles, and classify shapes by properties of their lines and angles. 1. Draw points, lines, line segments, rays, angles (right, acute, obtuse), and perpendicular and parallel lines. Identify these in two-dimensional figures. 2. Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines, or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. 3. Recognize a line of symmetry for a two-dimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry.</p> <p>Common Core State Standards for Mathematical Practice 3. Construct viable arguments and critique the reasoning of others. 5. Use appropriate tools strategically. 7. Look for and make use of structure.</p>	<p>Comments</p> <p>In this unit, 4.OA.C.5 includes repeated and growing shape patterns.¹⁰</p> <p>4.G.A.1 was first addressed in unit 10, and is addressed in its entirety in this unit to include perpendicular and parallel lines.</p> <p>The concepts in this unit lend themselves to using technology applications (MP.5).</p> <p>Students understand that geometric figures can be classified by analyzing various properties (MP.7) and justify their conclusions by using viable arguments (MP.3).</p>

¹⁰ For more information about possible patterns, see pages 30-31 in the Operations and Algebraic Thinking progressions document.

Sequenced Units for the Common Core State Standards in Mathematics Grade 4

Unit 14: Problem solving with whole numbers	Suggested number of days: 15
<p>This is a culminating unit in which students focus on problem solving in order to demonstrate fluency with the standard algorithms in addition and subtraction. They demonstrate computational fluency with all problem types.</p> <p>All standards in this unit have been addressed in prior units. These concepts require greater emphasis due to the depth of the ideas, the time they take to master, and/or their importance to future mathematics.</p>	
<p>Common Core State Standards for Mathematical Content</p> <p>Operations and Algebraic Thinking — 4.OA</p> <p>A. Use the four operations with whole numbers to solve problems.</p> <p>2. Multiply or divide to solve word problems involving multiplicative comparison, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem, distinguishing multiplicative comparison from additive comparison.¹</p> <p>NOTE: ¹See Glossary, Table 2.</p> <p>3. Solve multistep word problems posed with whole numbers and having whole-number answers using the four operations, including problems in which remainders must be interpreted. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.</p> <p>Number and Operations in Base Ten² — 4.NBT</p> <p>B. Use place value understanding and properties of operations to perform multi-digit arithmetic.</p> <p>4. Fluently add and subtract multi-digit whole numbers using the standard algorithm.</p> <p>5. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>6. Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.</p> <p>NOTE: ²Grade 4 expectations in this domain are limited to whole numbers less than or equal to 1,000,000.</p> <p>Common Core State Standards for Mathematical Practice</p> <p>2. Reason abstractly and quantitatively.</p> <p>8. Look for and express regularity in repeated reasoning.</p>	<p>Comments</p> <p>In demonstrating fluency, students explain and flexibly use properties of operations and place value to solve problems, looking for shortcuts and applying generalized strategies (MP.2, MP.8).</p>