Rigor

Task Handout, Algebra 1
Conceptual Understanding Task #1

Suppose $P$ and $Q$ give the sizes of two different animal populations, where $Q > P$. In (a)-(f), say which of the given pair of expressions is larger. Briefly explain your reasoning in terms of the two populations.

a. $P + Q$ and $2P$

b. $\frac{P}{P + Q}$ and $\frac{P + Q}{2}$

c. $(Q - P)/2$ and $Q - P/2$

d. $P + 50t$ and $Q + 50t$

e. $\frac{P}{P + Q}$ and 0.5

f. $\frac{P}{Q}$ and $\frac{Q}{P}$

Conceptual Understanding Task #2

Felicia notices what appears to be an interesting pattern between powers of 11 and powers of $x + 1$:

\[
\begin{align*}
11^0 &= 1 & (x + 1)^0 &= 1 \\
11^1 &= 11 & (x + 1)^1 &= x + 1 \\
11^2 &= 121 & (x + 1)^2 &= x^2 + 2x + 1
\end{align*}
\]

*The digits of the number $11^n$ are the same as the coefficients of the polynomial $(x + 1)^n$. Is this always true?*

a. Does this pattern continue for $n = 3$ and $n = 4$?

b. What is the answer to Felicia’s question?

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Conceptual Understanding Task #3

Suppose a friend tells you she paid a total of $16,368 for a car, and you'd like to know the car's list price (the price before taxes) so that you can compare prices at various dealers. Find the list price of the car if your friend bought the car in:

a. Arizona, where the sales tax is 6.6%.

b. New York, where the sales tax is 8.25%.

c. A state where the sales tax is $r$.

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Procedural Skills and Fluency Task #1

12. The expression $3x^2 - 33x - 180$ can be factored into the form $a(x + b)(x + c)$, where $a$, $b$, and $c$ are constants, to reveal the zeros of the function defined by the expression. What are the zeros of the function defined by $3x^2 - 33x - 180$?

Select all that apply.

- $-15$
- $-10$
- $-6$
- $-4$
- $4$
- $6$
- $10$
- $15$

Procedural Skills and Fluency Task #2

a. Below is a quadrilateral $ABCD$:

Show, by dividing $ABCD$ into triangles, that the sum of the interior angles is $360^\circ$:

$m(\angle A) + m(\angle B) + m(\angle C) + m(\angle D) = 360$.

b. Below is pentagon $ABCDE$:

Show that the sum of the interior angles is $540^\circ$:

$m(\angle A)+m(\angle B)+m(\angle C)+m(\angle D)+m(\angle E)=540$.

c. Suppose $P$ is a polygon with $n \geq 3$ sides and assume that all interior angles of $P$ measure less than $180$ degrees. Show that the sum of the measures of the interior angles of $P$ is

$(n-2)\times 180$ degrees.

Check that this formula gives the correct value for equilateral triangles and squares.

Procedural Skills and Fluency Task #3

A non-negative polynomial $f$ is a polynomial which never takes negative values, that is, $f(x) \geq 0$ for all real values of $x$.

a. Decide which of the following polynomials are non-negative:

\[
x^2 \quad x^2 - 1 \quad x^3 \quad 100000 - x^2 \quad mx + b
\]

In the last part, consider various possibilities for $m$ and $b$.

b. Show that if $g$ is a polynomial, then $g^2$ is a non-negative polynomial. Use this fact to generate some non-negative polynomials.

c. Are all of the coefficients of a non-negative polynomial necessarily positive?

d. Is there a non-negative polynomial which has all negative coefficients?

e. Find a non-negative polynomial which is not the square of another polynomial.

Application Task #1

A model airplane pilot is practicing flying her airplane in a big loop for an upcoming competition. At time $t = 0$ her airplane is at the bottom of the loop 100 feet above the ground. The loop is supposed to be a perfect circle, at its highest point the airplane is 400 feet above the ground, and it takes her 60 seconds to complete one loop. The airplane completes two loops and it is supposed to fly at a constant speed the entire time to score well in the competition.

a. The diagram below is a representation of the loop of the airplane. Draw in points that show where the airplane is every 15 seconds, starting at $t = 0$. Assume that the plane is going around the circle in the counterclockwise direction.

b. Make a table of values of the height of the airplane in feet above the ground as a function of time in seconds for the two loops.

c. Draw a graph of the height of the airplane during the two minute time interval on the axes below.

Application Task #2

A restaurant is open from 2 pm to 2 am on a certain day, and a maximum of 200 clients can fit inside. If \( f(t) \) is the number of clients in the restaurant \( t \) hours after 2 pm that day,

a. What is a reasonable domain for \( f \)?

b. What is a reasonable range for \( f \)?

Application Task #3

Jane wants to sell her Subaru Forester and does research online to find other cars for sale in her area. She checks on craigslist.com and finds 22 Subaru Foresters recently listed, along with their mileage (in miles), age (in years), and listed price (in dollars). (Collected on June 6th, 2012 for the San Francisco Bay Area.)

She examines the scatterplot of price versus age and determines that a linear model is appropriate. She finds the equation of the least squares regression equation:

\[
\text{predicted price} = 24,247.56 - 1482.06 \text{ age}.
\]

a. What variable is the explanatory (independent) variable and what are the units it is measured in? What variable is the response (dependent) variable and what are the units it is measured in?

b. What is the slope of the least squares regression line and what are its units?

c. Interpret the slope of the least squares regression line in the context of the problem, discussing what the slope tells you about how price and age are related. Use appropriate units in your answer.

d. What is the y-intercept of the least squares regression line? Interpret the y-intercept in the context of the problem, including appropriate units.

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