Standards INSTITUTE

Rigor in the Standards-Conceptual Understanding

Handout, High School

Rigor in the Standards

The *High School Publishers' Criteria* gives a high level description of rigor for grades 9 through 12, and while it is not exhaustive, it is meant to frame your thinking around rigor for this grade band. This "Rigor in the Standards" handout, and the examples contained within, should be used to discuss the meaning, intent, and themes of the major work for this grade band. Use this document as a resource during planning or professional learning opportunities to frame conversations around rigor within this grade band and to reflect on the instructional practices necessary to appropriately attend to rigor in content standards.

"To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor: (1) conceptual understanding, (2) procedural skill and fluency, and (3) applications. The word "rigor" isn't a code word for just one of these three; rather, it means equal
intensity in all three. The word "understand" is used in the Standards to set explicit expectations for conceptual understanding, and the phrase "real-world problems" and the star symbol (★) are used to set expectations and flag opportunities for applications and modeling. (Modeling is a Standard for Mathematical Practice as well as a content category in High School.) The High School content standards do not set explicit expectations for fluency, but fluency is important in high school mathematics." ***—High School Publishers' Criteria for the Common Core State Standards for Mathematics***

At UnboundEd, we've studied the state standards, spent time in classrooms, and looked at work done by other organizations to form an understanding of these three aspects of rigor that we think is most useful for educators to understand the standards and shift their practice. So while the words *understand*, *fluently*, and *real-world problems* do indicate the three aspects of rigor, they are not comprehensive. We've come to associate conceptual understanding with higher order thinking skills, working with multiple representations, and teaching more than just computational procedures. Procedural skills are about students accurately performing core functions required for grade-level mathematics; fluency is explicitly called for in certain standards and implies efficiency. Application can be thought of generally as problem solving, in real-world or mathematical contexts. For example, the words *recognize* or *compar*e can be used to indicate conceptual understanding, *count* can indicate procedural skill and fluency, and *solve addition and subtraction word problems* can be used to indicate application. Nevertheless, the example standards here that indicate an aspect of rigor should be used as examples and are not meant to be a checklist or keyword indicators.

Additional Aspects of the Rigor and Balance Criterion from the *High School Publishers' Criteria*:
(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be

practiced in the context of applications; and applications can build conceptual understanding.)
(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free unless explicitly taught.)

Conceptual Understanding

"Developing students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings. Materials amply feature high-quality conceptual problems and questions. This includes brief conceptual problems with low computational difficulty (e.g., 'What is the maximum value of the function $f(t) = 5 - t^2$?'); brief conceptual questions (e.g., 'Is $\sqrt{2}$ a polynomial? How about $\frac{1}{2}(x + \sqrt{2}) + \frac{1}{2}(-x + \sqrt{2})$)+?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships. [5] Classroom discussion about such problems can offer opportunities to engage in mathematical practices such as constructing and critiquing arguments (MP.3). In the materials, conceptual understanding is attended to most thoroughly in those places in the content standards where explicit expectations are set for understanding or interpreting. Such problems and activities center on fine-grained mathematical concepts, such as the correspondence between an equation and its graph, solving equations as a process of answering a question, analyzing a nonlinear equation $f(x) = g(x)$ by graphing f and g on a single set of axes, etc. Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards."
—High School Publishers' Criteria for the Common Core State Standards for Mathematics

The *High* School *Publishers' Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first, we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building conceptual understanding.

Conceptual Understanding	
Main goals:	**Effective instructional strategies:**
● Introduce concepts. ● Emphasize sensemaking instead of answer-getting. ● Uncover and unscramble common misconceptions.	○ **Discussion and reflection:** Students build their own understanding through experience, discussion, explaining, justifying, and/or reflection; teacher facilitates through questioning and making connections. ○ **Manipulatives and visual models:** Deepen knowledge of concepts before moving to abstract representations. ○ **Multiple representations:** Provide opportunities for students to experience and work between different representations of the same content (e.g., table, graph) ○ **Error analysis:** Target common misconceptions by determining if a mistake exists; explain the mistake.
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9, 2018	

The examples below are standards within high school grades that indicate conceptual understanding. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor in this grade band, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of the aspect of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been grayed.

<table>
<tr><th colspan="2">Language of the standards that indicates conceptual understanding:</th></tr>
<tr><td colspan="2">Explain
G-CO.B.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.</td></tr>
<tr><td>Rationale:</td><td>Addresses the conceptual understanding aspect of rigor because explaining why the triangle congruence criteria work requires deep understanding of congruence in terms of rigid motions. In G-CO.B.8, students use the definition of congruence to explain the criteria for triangle congruence.</td></tr>
<tr><td>Standards:</td><td>N-RN.A.1, S-IC.B.3</td></tr>
<tr><td>More to know:</td><td>A-REI.A.1 indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language:
A-REI.A.1: Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
Explaining each step in solving an equation and constructing a viable argument to justify the method of solving an equations demonstrates conceptual understanding.

A.SSE.B.3 indicates both conceptual understanding and procedural skills and fluency. Conceptual understanding is indicated in the standard by the language:
A.SSE.B.3: Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
Explaining how the properties of equivalent expressions are related demonstrates conceptual understanding.</td></tr>
</table>

Language of the standards that indicates conceptual understanding:	
Understand F-IF.A.1 **Understand** that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.	
Rationale:	Addresses the conceptual understanding aspect of rigor because students must understand, fundamentally, what a function is. In F-IF.A.1, students recognize that function notation represents the assignment of a domain to a particular range.
Standards:	A-REI.D.10, G-SRT.C.6
More to know:	A-APR.A.1 indicates both conceptual understanding and procedural skill and fluency. Conceptual understanding is indicated in the standard with the language: ● A-APR.A.1: ***Understand*** *that polynomials* ***form a system analogous*** *to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication;* *add, subtract, and multiply polynomials.* Conceptual understanding the analogous relationship of operating with integers and operating with polynomials is used to perform operations with polynomials.

<table>
<tr><th colspan="2">Language of the standards that indicates conceptual understanding:</th></tr>
<tr><td colspan="2">Interpret/Distinguish/Use...to decide
S-ID.C.7 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.</td></tr>
<tr><td>Rationale:</td><td>Addresses the conceptual understanding aspect of rigor because, rather than merely calculating slope, students must interpret the meaning of the different components of the representation in context. In S-ID.C.7, students interpret a linear model, including specifically determining the meaning of slope and the intercept within the context of the data.</td></tr>
<tr><td>Standards:</td><td>A-SSE.A.1, A-SSE.A.1.A, A-SSE.A.1.B, S-ID.C.9</td></tr>
<tr><td>More to know:</td><td>G-SRT.A.2 indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language:
G-SRT.A.2: Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
Using definitions to interpret two figures as being similar and then explaining the meaning of similarity for triangles demonstrates conceptual understanding.

F-IF.B.6 indicates both conceptual understanding and procedural skill and fluency. Conceptual understanding is indicated in the standard with the language:
F-IF.B.6: Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.*
Interpreting the average rate of change after calculating it demonstrates conceptual understanding.</td></tr>
</table>

Language of the standards that indicates conceptual understanding:	
Prove/Derive/Construct a Viable Argument/Verify G-SRT.B.4 **Prove** theorems about triangles. *Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.*	
Rationale:	Addresses the conceptual understanding aspect of rigor because, rather than merely finding missing angles or sides in triangle diagrams, students must employ the higher order, logical thinking associated with proofs. In G-SRT.B.4, students prove theorems about triangles.
Standards:	G-CO.C.9, G-CO.C.10, G-CO.C.11
More to know:	A-REI.B.4.A indicates both conceptual understanding and procedural skill and fluency. Conceptual understanding is indicated in the standard with the language: ● A-REI.B.4: Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2=q$ that has the same solutions. **Derive** the quadratic formula **from this form**. After using the completing the square algorithm, students derive the quadratic formula formula. This gives conceptual understanding of the formula that students will use later and requires higher level thinking beyond mere use of the formula to solve equations. .

Language of the standards that indicates conceptual understanding:	
Recognize / Identify F-IF.A.3 **Recognize** that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1*.	
Rationale:	Addresses conceptual understanding because students must deeply understand what functions are in order to recognize that sequences are functions; computation or problem solving are not emphasized here, only a focus on understanding the relationship between functions and sequences. In F-IF.A.3, students transfer their knowledge of functions to better understand sequences.
Standards:	A-SSE.A.2
More to know:	S-IC.B.3 indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language: ● S-IC.B.3: ***Recognize the purposes*** *of and differences among sample surveys, experiments, and observational studies;* ***explain*** *how randomization relates to each.* Transferring knowledge of randomization to statistical experiments to understand and explain the purposes and differences between them demonstrates conceptual understanding. A-APR.B.3 indicates both conceptual understanding and procedural skill and fluency. Conceptual understanding is indicated in the standard with the language: ● A-APR.B.3: ***Identify*** *zeros of polynomials when suitable factorizations are available,* *and use the zeros to construct a rough graph of the function defined by the polynomial.* Conceptual understanding of the relationship between zeros and factors of polynomials is required in order to identify zeros of polynomials by factoring; this moves beyond performing the basic procedure of factoring. The knowledge of zeros enhances the understanding of factorizations.