Standards INSTITUTE

Rigor in the Standards-Conceptual Understanding

Handout, Grades K-2

Rigor in the Standards

The *K–8 Publishers' Criteria* gives a high level description of rigor for grades K through 8, and while it is not exhaustive, it is meant to frame your thinking around rigor for this grade band. This "Rigor in the Standards" handout, and the examples contained within, should be used to discuss the meaning, intent, and themes of the major work for this grade band. Use this document as a resource during planning or professional learning opportunities to frame conversations around rigor within this grade band and to reflect on the instructional practices necessary to appropriately attend to rigor in content standards.

"To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word *understand* is used in the Standards to set explicit expectations for conceptual understanding, the word "*fluently"* is used to set explicit expectations for fluency, and the phrase "*real-world problems"* and the star symbol (*) is used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a content category in High School)." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

At UnboundEd, we've studied the state standards, spent time in classrooms, and looked at work done by other organizations to form an understanding of these three aspects of rigor that we think is most useful for educators to understand the standards and shift their practice. So while the words *understand*, *fluently*, and *real-world problems* do indicate the three aspects of rigor, they are not comprehensive. We've come to associate conceptual understanding with higher order thinking skills, working with multiple representations, teaching more than just computational procedures. Procedural skills are about students accurately performing core functions required for grade-level mathematics; fluency is explicitly called for in certain standards and implies efficiency. Application can be thought of generally as problem solving, in real-world or mathematical contexts. For example, the words *recognize* or *compare* can be used to indicate conceptual understanding, *count* can indicate procedural skill and fluency, and *solve addition and subtraction word problems* can be used to indicate application. Nevertheless, the example standards that indicate an aspect of rigor should be used as examples and are not meant to be a checklist or keyword indicators.

Additional Aspects of the Rigor and Balance Criterion from the *K–8 Publishers' Criteria*:
(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)

(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free unless explicitly taught.)

Conceptual Understanding

"Developing students' conceptual understanding of key mathematical concepts, especially where called for in specific content standards or cluster headings. Materials amply feature high-quality conceptual problems and questions that can serve as fertile conversation starters in a classroom if students are unable to answer them. This includes brief conceptual problems with low computational difficulty (e.g., 'Find a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$ '); brief conceptual questions (e.g., 'If the divisor does not change and the dividend increases, what happens to the quotient?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships. In the materials, conceptual understanding is not a generalized imperative applied with a broad brush, but is attended to most thoroughly in those places in the content standards where explicit expectations are set for *understanding* or *interpreting*. Such problems and activities include fine-grained mathematical concepts, such as place value, the whole-number product $a \times b$, the fraction $\frac{a}{b}$, the fraction product $(a/b) \times q$, expressions as records of calculations, solving equations as a process of answering a question, etc. (Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.)"
—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics

The *K–8 Publishers' Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first, we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building conceptual understanding.

Conceptual Understanding	
Main goals:	**Effective instructional strategies:**
● Introduce concepts. ● Emphasize sensemaking instead of answer-getting. ● Uncover and unscramble common misconceptions.	○ **Discussion and reflection:** Students build their own understanding through experience, discussion, explaining, justifying, and/or reflection; teacher facilitates through questioning and making connections. ○ **Manipulatives and visual models:** Deepen knowledge of concepts before moving to abstract representations. ○ **Multiple representations:** Provide opportunities for students to experience and work between different representations of the same content (e.g., table, graph). ○ **Error analysis:** Target common misconceptions by determining if a mistake exists; explain the mistake.
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9, 2018	

The examples below are standards within grades K–2 that indicate conceptual understanding. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor in this grade band, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of the aspect of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been grayed.

Language of the standards that indicates conceptual understanding:	
Compose and Decompose K.NBT.A.1 **Compose and decompose** numbers from 11 to 19 into ten ones and some further ones, e.g., **by using objects or drawings, and record each composition or decomposition by a drawing or equation (such as 18 = 10 + 8); understand that these numbers are composed of ten ones and one, two, three, four, five, six, seven, eight, or nine ones.**	
Rationale:	Addresses the conceptual understanding aspect of rigor by building students' foundation of place value. In K.NBT.A.1, students compose and decompose numbers using concrete objects or drawings and relate to drawings or equations to build conceptual understanding of tens and ones, setting the foundation for place value understanding.
Standards:	K.OA.A.3
More to know:	K.NBT.A.1 indicates conceptual understanding in multiple ways. Conceptual understanding is indicated through the composition and decomposition of teen numbers, use of concrete and pictorial representations, and in building place value understanding. 1.OA.C.6 indicates both conceptual understanding and procedural skills and fluency. Conceptual understanding is indicated in the standard with the language: ● 1.OA.C.6: Add and subtract within 20, **demonstrating fluency** for addition and subtraction within 10. ***Use strategies such as counting on; making ten*** *(e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);* ***decomposing a number leading to a ten*** *(e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9);* ***using the relationship between addition and subtraction*** *(e.g., knowing that 8 + 4 = 12, one knows 12 – 8 = 4); and* ***creating equivalent but easier or known sums*** *(e.g., adding 6 + 7 by creating the known equivalent 6 + 6 + 1 = 12 + 1 = 13).* Addition and subtraction within 20 is developed through conceptual understanding using the strategies described within the standard. Students use their conceptual understanding of composing and decomposing a ten, the relationship between addition and subtraction, and creating equivalent but easier known sums to add and subtract within 20 and to build fluency of addition and subtraction within 10. The conceptual strategies described in 1.OA.C.6 are extended to mental strategies in Grade 2. 2.OA.B.2 specifically calls out fluency with addition and subtraction within 20.

Language of the standards that indicates conceptual understanding:	
Compare 1.NBT.B.3 **Compare** two two-digit numbers based on the meaning of the tens and ones digits, recording the results of comparisons with the symbols >, =, and <.	
Rationale:	Addresses the conceptual understanding aspect of rigor by building students' understanding of place value. In 1.NBT.B.3, students compare two two-digit numbers using the place value of the digits within each number. Students have to understand the composition of two-digit numbers and what this composition means when related to another two-digit number.
Standards:	K.CC.C.7, 2.NBT.A.4

Language of the standards that indicates conceptual understanding:	
Represent K.OA.A.1 **Represent** addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), acting out situations, verbal explanations, expressions, or equations.	
Rationale:	Addresses the conceptual understanding aspect of rigor by building students' understanding of operations using representations. In K.OA.A.1, students represent addition as putting together and adding to and subtraction as taking apart and taking from in problem situations using a variety of concrete or pictorial representations.
Standards:	2.MD.B.6
More to know:	K.CC.A.3 and 1.NBT.A.1 indicate conceptual understanding and procedural skills and fluency. Conceptual understanding is indicated in the each of the standards by "representing a number of objects": ● K.CC.A.3: ***Write*** *numbers from 0 to 20.* ***Represent*** *a number of objects with a written numeral 0–20 (with 0 representing a count of no objects).* ● 1.NBT.A.1: ***Count*** *to 120, starting at any number less than 120. In this range,* ***read and write*** *numerals* ***and represent*** *a number of objects with a written numeral.* Procedural skills and fluency is indicated by the writing numbers and counting components of the standards.

Language of the standards that indicates conceptual understanding:	
Understand 2.NBT.A.1 **Understand** that the three digits of a three-digit number represent amounts of hundreds, tens, and ones; e.g., 706 equals 7 hundreds, 0 tens, and 6 ones.	
Rationale:	Addresses the conceptual understanding aspect of rigor by having students demonstrate understanding of place value. In 2.NBT.A.1, students understand place value in base ten through hundreds.
Standards:	K.CC.B.4.B, K.CC.B.4.C, 1.OA.B.4, 1.OA.D.7, 1.NBT.B.2, 2.NBT.A.1
More to know:	1.NBT.C.4 indicates both conceptual understanding and procedural skills and fluency. Conceptual understanding is indicated in the standard with the language: ● 1:NBT.C.4: Add within 100, including adding a two-digit number and a one-digit number, and adding a two-digit number and a multiple of 10, *using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategies to a written method and explain the reasoning used.* ***Understand*** *that in adding two-digit numbers, one adds tens and tens, ones and ones; and sometimes it is necessary to compose a ten.* Students must conceptually understand place value to add within 100 and to relate the addition strategies to the written method to understand why it works. Procedural skill is indicated by the addition of a two-digit and a one-digit number and the addition of a two-digit number and a multiple of 10, both within 100. The conceptual aspect of this standard supports building proficiency in the procedural skill of adding two-digit numbers using a written method. 2.NBT.B.7 indicates both conceptual understanding and procedural skills and fluency similarly to 1.NBT.C.4, where the conceptual aspect of this standard supports building proficiency in the procedural skill of using a written method to solve addition and subtraction problems within 1000. ● *2.NBT.B.7: Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method.* ***Understand*** *that in adding or subtracting three-digit numbers, one adds or subtracts hundreds*

	and hundreds, tens and tens, ones and ones; and sometimes it is necessary to compose or decompose tens or hundreds.