Standards INSTITUTE

Rigor in the Standards-
Conceptual Understanding

Handout, Grades 3-5

Rigor in the Standards

The *K–8 Publishers' Criteria* gives a high level description of rigor for grades K through 8, and while it is not exhaustive, it is meant to frame your thinking around rigor for this grade band. This "Rigor in the Standards" handout, and the examples contained within, should be used to discuss the meaning, intent, and themes of the major work for this grade band. Use this document as a resource during planning or professional learning opportunities to frame conversations around rigor within this grade band and to reflect on the instructional practices necessary to appropriately attend to rigor in content standards.

"To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word *understand* is used in the Standards to set explicit expectations for conceptual understanding, the word "*fluently"* is used to set explicit expectations for fluency, and the phrase "*real-world problems"* and the star symbol (*) is used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a content category in High School)." —***K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

At UnboundEd, we've studied the state standards, spent time in classrooms, and looked at work done by other organizations to form an understanding of these three aspects of rigor that we think is most useful for educators to understand the standards and shift their practice. So while the words *understand*, *fluently*, and *real-world problems* do indicate the three aspects of rigor, they are not comprehensive. We've come to associate conceptual understanding with higher order thinking skills, working with multiple representations, and teaching more than just computational procedures. Procedural skills are about students accurately performing core functions required for grade-level mathematics; fluency is explicitly called for in certain standards and implies efficiency. Application can be thought of generally as problem solving, in real-world or mathematical contexts. For example, the words *recognize* or *compar*e can be used to indicate conceptual understanding, *count* can indicate procedural skill and fluency, and *solve addition and subtraction word problems* can be used to indicate application. Nevertheless, the example standards that indicate an aspect of rigor should be used as examples and are not meant to be a checklist or keyword indicators.

Additional Aspects of the Rigor and Balance Criterion from the *K–8 Publishers' Criteria*:
(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)
(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free but must be explicitly taught.)

Conceptual Understanding

"Developing students' conceptual understanding of key mathematical concepts, where called for in specific content standards or cluster headings. Materials amply feature high-quality conceptual problems and questions that can serve as fertile conversation starters in a classroom if students are unable to answer them. This includes brief conceptual problems with low computational difficulty (e.g., 'Find a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$ '); brief conceptual questions (e.g., 'If the divisor does not change and the dividend increases, what happens to the quotient?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships. In the materials, conceptual understanding is not a generalized imperative applied with a broad brush, but is attended to most thoroughly in those places in the content standards where explicit expectations are set for *understanding* or *interpreting*. Such problems and activities include fine-grained mathematical concepts, such as place value, the whole-number product $a \times b$, the fraction $\frac{a}{b}$, the fraction product $(a/b) \times q$, expressions as records of calculations, solving equations as a process of answering a question, etc. (Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.)" —***K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

The *K–8 Publishers' Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building conceptual understanding.

Conceptual Understanding	
Main goals:	**Effective instructional strategies:**
● Introduce concepts. ● Emphasize sensemaking instead of answer-getting.	○ **Discussion and reflection:** Students build their own understanding through experience, discussion, explaining, justifying, and/or reflection; teacher facilitates through questioning and making connections. ○ **Manipulatives and visual models:** Deepen knowledge of concepts before moving to abstract representations. ○ **Multiple representations:** Provide opportunities for students to experience and work between different representations of the same content (e.g., table, graph). ○ **Error analysis:** Target common misconceptions by determining if a mistake exists; explain the mistake.

• Uncover and unscramble common misconceptions.	
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9, 2018	

The examples below are standards within grades 3–5 that indicate conceptual understanding. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor in this grade band, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of the aspect of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been grayed.

Language of the standards that indicates conceptual understanding:	
Understand and Represent 3.NF.A.2 **Understand** a fraction as a number on the number line; **represent** fractions on a number line diagram.	
Rationale:	Addresses the conceptual understanding aspect of rigor in multiple ways. In 3.NF.A.2, students must understand fractions as numbers on the number line and represent fractions on a number line; the standard explicitly names understanding, along with use of a visual representation.
Standards:	3.OA.B.6, 3.NF.A.1, 3.NF.A.3.A, 3.MD.C.5, 4.NF.B.3.A, 4.NF.B.4.A, 4.NF.B.4.B
More to know:	3.NF.A.2.A and 3.NF.A.2.B both address conceptual understanding because students represent fractions on a number line, including defining the interval that is the whole and partitioning it into equal parts and understanding the fractional size of each part, in support of conceptually understand fractions as numbers.

	3.OA.D.8 indicates both conceptual understanding and application. Conceptual understanding is indicated in the standard by the language: ● 3.OA.D.8 ***Solve two-step word problems*** *using the four operations.* ***Represent*** *these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.* Students have to be able to apply their understanding of the four operations when solving word problems that require them to represent the context using equations with a variable for the unknown and assess the reasonableness of their answers using mental strategies and estimation.

Language of the standards that indicates conceptual understanding:	
Recognize 5.NBT.A.1 **Recognize** that in a multi-digit number, a digit in one place represents 10 times as much as it represents in the place to its right and 1/10 of what it represents in the place to its left.	
Rationale:	Addresses the conceptual understanding aspect of rigor because recognizing relationships based on place value depends securely on deep understanding of the place value system. 5.NBT.A.1 requires students to recognize the unique relationship a digit has to its place and the base-ten system, noticing that the value of the digit decreases by a magnitude of 10 as the digit moves to the place to its right within the multi-digit number.
Standards:	3.MD.C.5, 4.NBT.A.1, 5.MD.C.5.C, 3.NF.B.3.B
More to know:	3.MD.C.5 indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language: ● 3.MD.C.5 ***Recognize*** *area as an attribute of plane figures and* ***understand*** *concepts of area measurement.* Students extend understanding of plane figures to include the concept of area and that the area of a plane figure can be measured. 5.MD.C.5.C indicates conceptual understanding, procedural skills and fluency, and application. Conceptual understanding is indicated in the standard by the language:

	• 5.MD.C.5.C ***Recognize*** *volume as additive.* ***Find*** *volumes of solid figures composed of two non-overlapping right rectangular prisms by adding the volumes of the non-overlapping parts, applying this technique to* ***solve real world problems.*** Procedural skills are indicated in the standard in students using a procedure (adding volumes of non-overlapping parts) to find the volume of solid figures. Application is called out in the standard specifically in using this procedure to find volumes of solid figures when solving real world problems.

Language of the standards that indicates conceptual understanding:	
Interpret 3.OA.A.2 **Interpret** whole-number quotients of whole numbers, e.g., **interpret** 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. For example, describe a context in which a number of shares or a number of groups can be expressed as 56 ÷ 8.	
Rationale:	Addresses the conceptual understanding aspect of rigor because interpreting quotients, when compared with finding or calculating quotients, indicates higher order thinking that must involve deep understanding of division. In 3.OA.A.2, students have to be able to interpret how a whole number is divided into equal shares or the number of groups created when sharing. Students build their understanding through explaining problem contexts where they are interpreting quotients of whole numbers.
Standards:	3.OA.A.1, 4.OA.A.1, 5.OA.A.2, 5.NF.B.3, 5.NF.B.4.A
More to know:	5.NF.B.3 indicates both conceptual understanding and application. Conceptual understanding is indicated in the standard by the language: • 5.NF.B.3 ***Interpret*** *a fraction as division of the numerator by the denominator (a/b = a ÷ b).* ***Solve word problems*** *involving division of whole numbers leading to answers in the form of fractions or mixed numbers, e.g., by* ***using visual fraction models or equations to represent the problem****. For example,* ***interpret 3/4 as the result of dividing 3 by 4****, noting that 3/4 multiplied by 4 equals 3, and that when 3 wholes are shared equally among 4 people each person has a share of size 3/4. If 9 people want to share a 50-pound sack of rice equally by weight, how many pounds of rice should each person get? Between what two whole numbers does your answer lie?*

	The application aspect of rigor is indicated through the standard’s description of “solving word problems involving division of whole numbers.”

Language of the standards that indicates conceptual understanding:	
Compare 5.NBT.A.3.B **Compare** two decimals to thousandths **based on meanings** of the digits in each place, using >, =, and < symbols to record the results of comparisons.	
Rationale:	Addresses the conceptual understanding aspect of rigor because students have to be able understand the place value system. In 5.NBT.A.3.B, students use understanding of how place value gives a digit its value in order to make comparisons. Additionally, the standard specifically mentions comparing “based on meanings,” which implies that students must logically justify their comparisons, which requires higher order thinking beyond computation or procedural methods.
Standards:	3.NF.A.3.D, 4.NF.A.2, 4.NF.C.7, 5.NF.B.5.A
More to know:	3.NF.A.3.D indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language: ● 3.NF.A.3.D ***Compare*** *two fractions with the same numerator or the same denominator by reasoning about their size.* ***Recognize*** *that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, =, or <, and* ***justify the conclusions, e.g. by using a visual fraction model****.* 4.NF.A.2 also indicates conceptual understanding in multiple ways and is similar to 3.NF.A.3.D in that it indicates conceptual understanding through comparison, recognizing the validity of comparisons, and justifying the conclusion of the comparison. Students in grade 3 are developing an understanding of fractions as numbers, and students in grade 4 are extending their understanding of fraction equivalence and ordering.

Language of the standards that indicate conceptual understanding	
Explain 4.NF.A.1 **Explain** why a fraction a/b is equivalent to a fraction $(n \times a)/(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.	
Rationale:	Addresses the conceptual understanding aspect of rigor because deep understanding is required for students to be able to explain fraction equivalence; this standard moves beyond mere computation or identification of equivalent fractions. Students have to be able to visually represent the equivalent fractions and specify the differences amongst the parts in order to explain why the fractions are equivalent.
Standards:	3.NF.A.3, 5.NBT.A.2
More to know:	5.NF.B.5.B indicates conceptual understanding in multiple ways. Conceptual understanding is indicated in the standard by the language: ● 5.NF.B.5.B: ***Explaining*** *why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case);* ***explaining*** *why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and* ***relating the principle of fraction equivalence*** $a/b = (n \times a)/(n \times b)$ ***to the effect of multiplying*** a/b *by 1.* In 5.NF.B.5, students interpret multiplication as scaling by extending their previous understandings of multiplication and division to multiply and divide fractions. Beyond this interpreting and explaining, students explain why multiplying a given number by a fraction greater than 1 results in a product greater than the given number and explain why multiplying a given number by a fraction less than 1 results in a product smaller than the given number.

Language of the standards that indicates conceptual understanding:	
Relate 3.MD.C.7 **Relate** area to the operations of multiplication and addition.	
Rationale:	Addresses the conceptual understanding aspect of rigor because, in contrast to just computing products or sums, students need

	deeper understanding to be able to connect the concept of area to these operations. Students extend their understanding of these operations in order to relate them to area.
Standards:	5.MD.C.5
More to know:	5.MD.C.5 indicates both conceptual understanding and application. Conceptual understanding is indicated in the standard by the language: ● 5.MD.C.5: ***Relate*** *volume to the operations of multiplication and addition and* ***solve real world and mathematical problems*** *involving volume.* In 5.MD.C.5 students apply their understanding of volume and its relationship to the multiplication and addition to solve real-world and mathematical problems.

Language of the standards that indicates conceptual understanding:	
4.NBT.A.3 **Use place value understanding to round** multi-digit whole numbers, less than or equal to 1,000,000, to any place.	
Rationale:	Addresses the conceptual understanding aspect of rigor because understanding the place value system is explicitly named in the standard. In 4.NBT.A.3, students generalize their place value understanding when rounding multi-digit whole numbers.
Standards:	5.NBT.A.4, 3.NBT.A.1
More to know:	3.NBT.A.1 is considered Additional work, but it is foundational for the two rounding standards in grades 4 and 5, both of which are Major work.