Standards INSTITUTE

Rigor in the Standards-
Procedural Skills and Fluency

Handout, Grades 3-5

Rigor in the Standards

The *K–8 Publishers' Criteria* gives a high level description of rigor for grades K through 8, and while it is not exhaustive, it is meant to frame your thinking around rigor for this grade band. This "Rigor in the Standards" handout, and the examples contained within, should be used to discuss the meaning, intent, and themes of the major work for this grade band. Use this document as a resource during planning or professional learning opportunities to frame conversations around rigor within this grade band and to reflect on the instructional practices necessary to appropriately attend to rigor in content standards.

"To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word *understand* is used in the Standards to set explicit expectations for conceptual understanding, the word "*fluently"* is used to set explicit expectations for fluency, and the phrase "*real-world problems"* and the star symbol (*) is used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a content category in High School)." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

At UnboundEd, we've studied the state standards, spent time in classrooms, and looked at work done by other organizations to form an understanding of these three aspects of rigor that we think is most useful for educators to understand the standards and shift their practice. So while the words *understand*, *fluently*, and *real-world problems* do indicate the three aspects of rigor, they are not comprehensive. We've come to associate conceptual understanding with higher order thinking skills, working with multiple representations, and teaching more than just computational procedures. Procedural skills are about students accurately performing core functions required for grade-level mathematics; fluency is explicitly called for in certain standards and implies efficiency. Application can be thought of generally as problem solving, in real-world or mathematical contexts. For example, the words *recognize* or *compar*e can be used to indicate conceptual understanding, *count* can indicate procedural skill and fluency, and *solve addition and subtraction word problems* can be used to indicate application. Nevertheless, the example standards here that indicate an aspect of rigor should be used as examples and are not meant to be a checklist or keyword indicators.

Additional Aspects of the Rigor and Balance Criterion from the *K–8 Publishers' Criteria*:
(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)
(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free but must be explicitly taught.)

Procedural Skills and Fluency

"Giving attention throughout the year to individual standards that set an expectation of fluency. The Standards are explicit where fluency is expected. Materials in grades K–6 help students make steady progress throughout the year toward fluent (accurate and reasonably fast) computation, including knowing single-digit products and sums from memory (see, e.g., 2.OA.B.2 and 3.OA.C.7). Progress toward these goals is interwoven with students' developing conceptual understanding of the operations in question. Manipulatives and concrete representations such as diagrams that enhance conceptual understanding are closely connected to the written and symbolic methods to which they refer (see, e.g., 1.NBT). As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable—e.g., the sum $698 + 240$ or the system $x + y = 1$, $2x + 2y = 3$ —as well as an ample number of generic cases so that students can learn and practice efficient algorithms (e.g., the sum $8767 + 2286$). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks. Materials do not make fluency a generalized imperative to be applied with a broad brush, but attend most thoroughly to those places in the content standards where explicit expectations are set for fluency. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8)." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

The *K–8 Publishers' Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building procedural skills and fluency.

Procedural Skills and Fluency	
Main goals:	**Effective instructional strategies:**
• Learn or develop algorithms.	○ **Connect procedures to conceptual understanding:** Link algorithms to concepts, help students understand the "why" behind the procedure. ○ **Explicit instruction:** I Do, We Do, You Do, teacher "Think Aloud," or teacher modeling. ○ **Practice:** Spiraled or distributed practice with consistent teacher feedback to lead to fluency.

• Execute procedures accurately and efficiently. • Learn how to use models or tools.	

Source: **Achievement Network**
https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf
Retrieved Nov. 9, 2018

The examples below are standards within grades 3–5 that indicate procedural skills and fluency. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been grayed.

Language of the standards that indicates procedural skill and fluency:	
3.OA.C.7 **Fluently multiply and divide** within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, **know from memory all products of two one-digit numbers**.	
Rationale:	Addresses the procedural skill and fluency aspect of rigor because fluency is specifically called out in the standard. In 3.OA.C.7 students fluently multiply and divide within 100, students must also memorize the products of two one-digit numbers. The use of strategies such as the relationship between multiplication and division or properties of operations to fluently multiply and divide is rooted in students' understanding of multiplication and division.
Standards:	4.NBT.B.4, 5.NBT.B.5
More to know:	In 5.NBT.B.7, fluency is not directly called out. However, this standard indicates procedural skill and is rooted in conceptual understanding similarly to 3.OA.C.7 in the use of strategies including the relationship between operations and properties of

	operations to support computation. The language in the standard that indicates procedural skill and fluency: ● 5.NBT.B.7: ***Add, subtract, multiply, and divide decimals to hundredths****, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.* Students perform operations with multi-digit whole numbers and with decimals to hundredths indicating procedural skill. This standard also addresses the conceptual understanding aspect of rigor because students use concrete models or drawings and strategies based on place value and properties of operations as strategies to support their computation. They also relate these strategies to a written method and explain the reasoning used.

Language of the standards that indicates procedural skills and fluency:	
4.NF.C.5 **Express** a fraction with denominator 10 as an equivalent fraction with denominator 100, and use this technique to add two fractions with respective denominators 10 and 100. For example, express 3/10 as 30/100, and add 3/10 + 4/100 = 34/100.	
Rationale:	Addresses the procedural skills and fluency aspect of rigor because students have to use a procedure efficiently to meet the benchmark of the standard. In 4.NF.C.5, students create equivalent fractions when the denominator is 10 or 100, then add the two fractions after making common denominators. Creating equivalent fractions is rooted in students' understanding of fractions as numbers.
Standards:	3.NF.A.3.C, 4.MD.A.1
More to know:	3.NF.A.3.C indicates procedural skill and fluency and conceptual understanding. The language in the standards that indicates procedural skill and fluency: ● 3.NF.A.3.C: ***Express*** *whole numbers as fractions, and* ***recognize*** *fractions that are equivalent to whole numbers. Examples: Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.* 3.NF.A.3.C indicates conceptual understanding by requiring students to recognize fractions that are equivalent to whole numbers. The implication here is that the recognition occurs with automaticity, through the work of the standard.

Language of the standards that indicates procedural skill and fluency:	
3.MD.C.6 **Measure** areas **by counting** unit squares (square cm, square m, square in, square ft, and improvised units.)	
Rationale:	Addresses the procedural skills and fluency aspect of rigor because it requires students to perform mathematical procedures: measuring and counting. In 3.MD.C.6, students use the procedural skill of counting, decomposing, and recomposing unit squares in order to measure the area of figures.
Standards:	3.MD.A.2, 5.MD.C.4
More to know:	3.MD.A.2 indicates conceptual understanding, procedural skill in fluency, and application. The language in the standard that indicates procedural skill and fluency: ● 3.MD.A.2: ***Measure*** *and* estimate *liquid volumes and masses of objects using standard units of grams (g), kilograms (kg), and liters (l).* ***Add, subtract, multiply, or divide*** *to* ***solve one-step word problems*** *involving masses or volumes that are given in the same units, e.g., by using drawings (such as a beaker with a measurement scale) to represent the problem.* Conceptual understanding is indicated in 3.MD.A.2 in students estimating liquid volumes and masses, and in the use of drawings to represent the problem; these indicate higher order thinking and use of multiple representations. Application is indicated in the standard in students solving one-step word problems.