

# Rigor in the Standards- Procedural Skills and Fluency

Handout, High School

# Rigor in the Standards

The *High School Publishers' Criteria* gives a high level description of rigor for grades 9 through 12, and while it is not exhaustive, it is meant to frame your thinking around rigor for this grade band. This “Rigor in the Standards” handout, and the examples contained within, should be used to discuss the meaning, intent, and themes of the major work for this grade band. Use this document as a resource during planning or professional learning opportunities to frame conversations around rigor within this grade band and to reflect on the instructional practices necessary to appropriately attend to rigor in content standards.

“To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor: (1) conceptual understanding, (2) procedural skill and fluency, and (3) applications. The word “rigor” isn’t a code word for just one of these three; rather, it means equal intensity in all three. The word “understand” is used in the Standards to set explicit expectations for conceptual understanding, and the phrase “real-world problems” and the star symbol (★) are used to set expectations and flag opportunities for applications and modeling. (Modeling is a Standard for Mathematical Practice as well as a content category in High School.) The High School content standards do not set explicit expectations for fluency, but fluency is important in high school mathematics.” —*High School Publishers' Criteria for the Common Core State Standards for Mathematics*

At UnboundEd, we’ve studied the state standards, spent time in classrooms, and looked at work done by other organizations to form an understanding of these three aspects of rigor that we think is most useful for educators to understand the standards and shift their practice. So while the words *understand*, *fluently*, and *real-world problems* do indicate the three aspects of rigor, they are not comprehensive. We’ve come to associate conceptual understanding with higher order thinking skills, working with multiple representations, and teaching more than just computational procedures. Procedural skills are about students accurately performing core functions required for grade-level mathematics; fluency is explicitly called for in certain standards and implies efficiency. Application can be thought of generally as problem solving, in real-world or mathematical contexts. For example, the words *recognize* or *compare* can be used to indicate conceptual understanding, *count* can indicate procedural skill and fluency, and *solve addition and subtraction word problems* can be used to indicate application. Nevertheless, the example standards here that indicate an aspect of rigor should be used as examples and are not meant to be a checklist or keyword indicators.

## **Additional Aspects of the Rigor and Balance Criterion from the *High School Publishers' Criteria*:**

(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)

(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free but must be explicitly taught.)

# Procedural Skills and Fluency

**“Giving attention throughout the year to procedural skill and fluency.** In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8). Progress toward procedural skill and fluency is interwoven with students’ developing conceptual understanding of the operations in question. Manipulatives and concrete representations are connected to the written and symbolic methods to which they refer. As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable, as in solving  $(3x - 2)^2 = 6x - 4$ , as well as an ample number of generic cases so that students can learn and practice efficient and general methods (e.g., solving  $c + 8 - c^2 = 3(c - 1)^2 - 5$ ). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks.” *High School Publishers’ Criteria for the Common Core State Standards for Mathematics*

The *High School Publishers’ Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first, we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building procedural skills and fluency.

Procedural Skills and Fluency	
<b>Main goals:</b>	<b>Effective instructional strategies:</b>
<ul style="list-style-type: none"> <li>● Learn or develop algorithms.</li> <li>● Execute procedures accurately and efficiently.</li> </ul>	<ul style="list-style-type: none"> <li>○ <b>Connect procedures to conceptual understanding:</b> Link algorithms to concepts, help students understand the “why” behind the procedure.</li> <li>○ <b>Explicit instruction:</b> I Do, We Do, You Do, teacher “Think Aloud,” or teacher modeling.</li> <li>○ <b>Practice:</b> Spiraled or distributed practice with consistent teacher feedback to lead to fluency.</li> </ul>

<ul style="list-style-type: none"> <li>Learn how to use models or tools.</li> </ul>	
<p>Source: <b>Achievement Network</b>  <a href="https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf">https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf</a>  Retrieved Nov. 9, 2018</p>	

The examples below are standards within high school grades that indicate procedural skills and fluency. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been *grayed*.

Language of the standards that indicates procedural skills and fluency:	
<p><b>Solve/Find</b>  A-REI.B.3 <b>Solve</b> linear equations and inequalities in one variable, including equations with coefficients represented by letters.</p>	
Rationale:	Addresses procedural skills and fluency because students are expected to perform algebraic manipulations in order to solve equations and inequalities. In A-REI.B.3, students are expected to be fluent in solving various types of linear equations and inequalities in one variable.
Standards:	A-REI.B.4, G-GPE.B.6
More to know:	<p>A-REI.B.4.B indicates both procedural skill and fluency and conceptual understanding. Procedural skill and fluency is indicated in the standard with the language:</p> <ul style="list-style-type: none"> <li>A-REI.B.4.B: <b>Solve quadratic equations by inspection</b> (e.g., for <math>x^2 = 49</math> ), <b>taking square roots, completing the square, the quadratic formula and factoring</b>, as appropriate to the initial form of the equation. <i>Recognize when the quadratic formula gives complex solutions and write them as <math>a \pm bi</math> for real numbers <math>a</math> and <math>b</math>.</i></li> </ul> <p>Students solve quadratic equations with various methods that require algebraic manipulations and computational techniques.</p>

Language of the standards that indicates procedural skills and fluency:

**Calculate / Evaluate / Compute / Graph/Make**

*G-GPE.B.7* Use coordinates to **compute** perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.\*

Rationale:

Addresses procedural skills and fluency because students use procedures to compute a particular value(s). In *G-GPE.B.7*, students use the coordinate geometry to compute the perimeters and areas of geometric figures.

Standards:

A-REI.D.12, G-CO.D.12

Language of the standards that indicates procedural skills and fluency:

**Write/Rewrite/Arrange/Rearrange/**

*N-RN.A.2* **Rewrite** expressions involving radicals and rational exponents using the properties of exponents.

Rationale:

Addresses procedural skill and fluency because students use the exponent properties to symbolically manipulate expressions, including performing core operations like multiplication. In *N-RN.A.2* students use structure from properties of exponents to rewrite expressions with radicals and rational exponents.

Standards:

A-SSE.B.3.A, A-SSE.B.3.B, A-SSE.B.3.C, A.CED.A.4