Standards INSTITUTE

Rigor in the Standards-Conceptual Understanding

Handout, Grades 6-8

Rigor in the Standards

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"To help students meet the expectations of the Standards, educators will need to pursue, with equal intensity, three aspects of rigor in the major work of each grade: conceptual understanding, procedural skill and fluency, and applications. The word *understand* is used in the Standards to set explicit expectations for conceptual understanding, the word "*fluently"* is used to set explicit expectations for fluency, and the phrase "*real-world problems"* and the star symbol (*) is used to set expectations and flag opportunities for applications and modeling (which is a Standard for Mathematical Practice as well as a content category in High School)." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

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Additional Aspects of the Rigor and Balance Criterion from the *K–8 Publishers' Criteria*:
(1) The three aspects of rigor are not always separate in materials. (Conceptual understanding needs to underpin fluency work; fluency can be practiced in the context of applications; and applications can build conceptual understanding.)
(2) Nor are the three aspects of rigor always together in materials. (Fluency requires dedicated practice to that end. Rich applications cannot always

be shoehorned into the mathematical topic of the day. And conceptual understanding will not come along for free but must be explicitly taught.)

Conceptual Understanding

"Develop students' conceptual understanding of key mathematical concepts, where called for in specific content standards or cluster headings. Materials amply feature high-quality conceptual problems and questions that can serve as fertile conversation starters in a classroom if students are unable to answer them. This includes brief conceptual problems with low computational difficulty (e.g., 'Find a number greater than $\frac{1}{5}$ and less than $\frac{1}{4}$ '); brief conceptual questions (e.g., 'If the divisor does not change and the dividend increases, what happens to the quotient?'); and problems that involve identifying correspondences across different mathematical representations of quantitative relationships. In the materials, conceptual understanding is not a generalized imperative applied with a broad brush, but is attended to most thoroughly in those places in the content standards where explicit expectations are set for *understanding* or *interpreting*. Such problems and activities include fine-grained mathematical concepts, such as place value, the whole-number product $a \times b$, the fraction $\frac{a}{b}$, the fraction product $(a/b) \times q$, expressions as records of calculations, solving equations as a process of answering a question, etc. (Conceptual understanding of key mathematical concepts is thus distinct from applications or fluency work, and these three aspects of rigor must be balanced as indicated in the Standards.)" ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

The *K–8 Publishers' Criteria* sets expectations for materials to reflect the appropriate aspect of rigor called for in the Standards. In order to ensure instruction reflects the appropriate aspect of rigor, first we must unpack what rigor looks like in the standards and how instruction might reflect this aspect of rigor. The table below identifies the main goal and effective instructional strategies for building conceptual understanding.

Conceptual Understanding	
Main goals:	**Effective instructional strategies:**
● Introduce concepts. ● Emphasize sensemaking instead of answer-getting. ● Uncover and unscramble common misconceptions.	○ **Discussion and reflection:** Students build their own understanding through experience, discussion, explaining, justifying, and/or reflection; teacher facilitates through questioning and making connections. ○ **Manipulatives and visual models:** Deepen knowledge of concepts before moving to abstract representations. ○ **Multiple representations:** Provide opportunities for students to experience and work between different representations of the same content (e.g., table, graph). ○ **Error analysis:** Target common misconceptions by determining if a mistake exists; explain the mistake.
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9, 2018	

The examples below are standards within grades 6–8 that indicate conceptual understanding. Each example provided highlights language in the standard that indicates the aspect of rigor, rationale for why this standard indicates the aspect of rigor, other standards that similarly reflect the aspect of rigor in this grade band, and additional information that helps to articulate the nuance of the Standards and helps to paint a more complete picture of the aspect of rigor for this grade band. Language in the standard that reflects a different aspect of rigor than the one being highlighted has been grayed.

Language of the standards that indicates conceptual understanding:	
Interpret 8.F.A.3 **Interpret** the equation *y = mx + b* as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. *For example, the function A = s^2 giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line*.	
Rationale:	Addresses the conceptual understanding aspect of rigor because students need to deeply understand what a linear function is, and also to move beyond just the procedures involved with graphing a linear function. In 8.F.A.3, students need to understand what a linear function is, that its graph is a straight line, and that the equation *y = mx + b* represents one. Developing these understandings allows student to be able to give non-examples.
Other Standards in this grade band:	6.NS.C.7.A
More to know:	8.EE.B.5 indicates both conceptual understanding and application. Conceptual understanding is indicated in the standard with the language: ● 8.EE.B.5: Graph proportional relationships, **interpreting the unit rate as the slope of the graph**. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed. This standard also addresses the application aspect of rigor because students graph, interpret and compare in the context of real-world problem solving with proportional relationships.

Language of the standards that indicates conceptual understanding:	
Understand 6.RP.A.1 **Understand** the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	
Rationale:	Addresses the conceptual understanding aspect of rigor because the standard clearly states an expectation for students to be able to understand the concept of a ratio. In 6.RP.A.1, students need to understand that a ratio relates two quantities and know how to use ratio language to describe that relationship, using phrases such as *for every...there are*.
Standards:	6.RP.A.2, 6.NS.C.5, 6.NS.C.6, 6.NS.C.7, 7.EE.A.2, 8.EE.C.8.A, 8.F.A.1, 8.G.A.2, 8.G.A.4
More to know:	6.EE.B.5: This addresses the conceptual understanding and procedural aspects of rigor. Conceptual understanding is indicated in the standard with the language: ● 6.EE.B.5: **Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true?** Use substitution to determine whether a given number in a specified set makes an equation or inequality true. This standard also addresses the procedural aspect of rigor because students need to use the strategy of substitution to determine the answer.

Language of the standards that indicates conceptual understanding:	
Identify 6.EE.A.4 **Identify** when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). *For example, the expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.*	
Rationale:	Addresses the conceptual understanding aspect of rigor because students need to use higher order thinking to reason about when expressions are equivalent; this is in contrast to mere evaluation of expressions using substitution. In 6.EE.A.4, students need to understand what it means for two expressions to be equivalent before they can identify which expressions are equivalent. In this standard, expressions are equivalent when they name the same number regardless of which value is substituted into them.
Standards:	6.EE.A.2.B
More to know	7.RP.A.2.B: This addresses the conceptual and application aspects of rigor. Conceptual understanding is indicated in the standard with the language: ● 7.RP.A.2.B: **Identify** the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships. The standard also addresses the application aspect of rigor because all of the identifying is done in the context of modeling; the representations named in the standard (tables, graphs, etc.) are all used to model real-world proportional relationships.

Language of the standards that indicates conceptual understanding:	
Decide 7.RP.A.2.A **Decide** whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.	
Rationale:	Addresses the conceptual understanding aspect of rigor because students need to be able to use understanding of the concept of a proportional relationship in order to "decide," as described by the standard. In 7.RP.A.2.A, students need to understand what a proportional relationship is and how the relationship is modeled in a ratio table and/or graph before they can test a ratio table or graph for proportionality.

Language of the standards that indicates conceptual understanding:	
Explain 7.RP.A.2.D **Explain** what a point (*x*, *y*) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, *r*) where r is the unit rate.	
Rationale:	Addresses the conceptual understanding aspect of rigor because, in contrast to mere procedural plotting or graphing, students must employ higher order thinking in order to explain how an abstract representation relates to a real-world context. In 7.RP.A.2.D, students need to understand what a proportional relationship means within a context and be able to explain what the points (0,0) and the unit rate means within that context.
Standards:	8.EE.B.6, 8.G.B.6

Language of the standards that indicates conceptual understanding:	
Analyze 8.EE.C.8 **Analyze** and solve pairs of simultaneous linear equations.	
Rationale:	Addresses the conceptual understanding aspect of rigor because students need to move beyond only solving to "analyze," which implies higher order thinking and deep understanding of systems. In 8.EE.C.8, students analyze a pair of linear equations and then use that information to solve the problem. This standard also addresses the procedural skills and fluency aspect or rigor because students need to solve pairs of linear equations algebraically, graphically, and by inspection; they need to be able to perform the algebraic manipulations required to solve systems.

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Rigor in the Standards-Application

Handout, Grades 6-8

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Application

"Allowing teachers and students using the materials as designed to spend sufficient time working with engaging applications, without losing focus on the major work of each grade. Materials in grades K–8 include an ample number of single-step and multi-step contextual problems that develop the mathematics of the grade, afford opportunities for practice, and engage students in problem solving. Materials for grades 6–8 also include problems in which students must make their own assumptions or simplifications in order to model a situation mathematically. Applications take the form of problems to be worked on individually as well as classroom activities centered on application scenarios. Materials attend thoroughly to those places in the content standards where expectations for multi-step and real-world problems are explicit. Applications in the materials draw only on content knowledge and skills specified in the content standards, with particular stress on applying major work, and a preference for the more fundamental techniques from additional and supporting work. Modeling builds slowly across K–8, and applications are relatively simple in earlier grades. Problems and activities are grade-level appropriate, with a sensible tradeoff between the sophistication of the problem and the difficulty or newness of the content knowledge the student is expected to bring to bear." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

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Application	
Main goals:	**Effective instructional strategies:**
● Apply skills and understandings to new situations, other subject areas, real-world and problem-solving situations.	o **Problem-solving opportunities:** Provide time for student to work on tasks independently, with a partner, or in small groups with consistent teacher feedback. o **Share multiple solution methods:** Facilitate classroom discussions where students share, explain, and justify a variety of problem solving strategies and/or solutions. o **Intentionally integrate content:** Provide learning opportunities for students to apply their knowledge of multiple standards, clusters, or domains.
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9, 2018	

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Language of the standards that indicates application:	
Solve...problems 6.RP.A.3.B **Solve** unit rate **problems** including those involving unit pricing and constant speed. ***For example,*** *if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*	
Rationale:	Addresses the application aspect of rigor because students need to able to solve a real-world scenario using ratio reasoning. In 6.RP.A.3.B, students solve unit rate problems in the context of unit pricing and constant speed.
Standards:	7.EE.B.3, 7.RP.A.3, 8.EE.C.8.C

Language of the standards that indicates application:	
Real-world problems 6.EE.C.9 **Use variables** to represent two quantities **in a real-world problem** that change in relationship to one another; **write an equation** to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. **Analyze** the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. **For example**, in a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.	
Rationale:	Addresses the application aspect of rigor because students need to able to solve a real-world problem using the conceptual knowledge they have previously developed. In 6.EE.C.9, students use their understanding of the relationship between dependent and independent variables, equations, and graphs to work problems such as constant speed.
Standards:	7.EE.B.3

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Rigor in the Standards-Procedural Skills and Fluency

Handout, Grades 6-8

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Procedural Skills and Fluency

"Giving attention throughout the year to individual standards that set an expectation of fluency. The Standards are explicit where fluency is expected. Materials in grades K–6 help students make steady progress throughout the year toward fluent (accurate and reasonably fast) computation, including knowing single-digit products and sums from memory (see, e.g., 2.OA.B.2 and 3.OA.C.7). Progress toward these goals is interwoven with students' developing conceptual understanding of the operations in question. Manipulatives and concrete representations such as diagrams that enhance conceptual understanding are closely connected to the written and symbolic methods to which they refer (see, e.g., 1.NBT). As well, purely procedural problems and exercises are present. These include cases in which opportunistic strategies are valuable—e.g., the sum $698 + 240$ or the system $x + y = 1$, $2x + 2y = 3$ —as well as an ample number of generic cases so that students can learn and practice efficient algorithms (e.g., the sum $8767 + 2286$). Methods and algorithms are general and based on principles of mathematics, not mnemonics or tricks. Materials do not make fluency a generalized imperative to be applied with a broad brush, but attend most thoroughly to those places in the content standards where explicit expectations are set for fluency. In higher grades, algebra is the language of much of mathematics. Like learning any language, we learn by using it. Sufficient practice with algebraic operations is provided so as to make realistic the attainment of the Standards as a whole; for example, fluency in algebra can help students get past the need to manage computational details so that they can observe structure (MP.7) and express regularity in repeated reasoning (MP.8)." ***—K–8 Publishers' Criteria for the Common Core State Standards for Mathematics***

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Procedural Skills and Fluency	
Main goals:	**Effective instructional strategies:**
● Learn or develop algorithms ● Execute procedures accurately and efficiently ● Learn how to use models or tools	○ **Connect procedures to conceptual understanding:** Link algorithms to concepts, help students understand the "why" behind the procedure. ○ **Explicit instruction:** I Do, We Do, You Do, teacher "Think Aloud," or teacher modeling. ○ **Practice:** Spiraled or distributed practice with consistent teacher feedback to lead to fluency.
Source: **Achievement Network** https://static1.squarespace.com/static/5321dc4ae4b0c72ad0ceedfe/t/59c4179537c5811bd8d9000c/1506023318140/Instructional+Approaches+for+Math+Rigor.pdf Retrieved Nov. 9,2018	

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Language of the standards that indicates procedural skills and fluency:	
Fluently 6.NS.B.2 **Fluently** divide multi-digit numbers using the standard algorithm.	
Rationale:	Addresses the procedural skills and fluency aspect of rigor because it requires fluency with an algorithm for division. In 6.NS.B.2, students must use the standard algorithm for dividing multi-digit numbers.
Standards:	6.NS.B.3

Language of the standards that indicates procedural skills and fluency:	
Apply properties of ... 7.EE.A.1 **Apply properties of operations** as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.	
Rationale:	Addresses the procedural skills and fluency aspect of rigor because it requires students to perform operations and other procedures with linear expressions. In 7.EE.A.1, students must be able to easily access the properties of operations in order to manipulate expressions.
Standards:	7.NS.A.1.D, 7.NS.A.2.C
More to know:	8.EE.A.1 This addresses the conceptual understanding and procedural skills and fluency aspects of rigor. Procedural skills and fluency are indicated in the standard with the language: ● 8.EE.A.1: **Know** and **apply the properties of integer exponents to generate equivalent numerical expressions.** For example, $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$. The verb "know" indicates conceptual understanding of the properties is required as well.